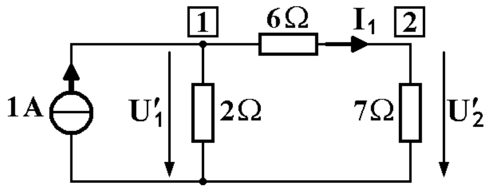
SOLUTION SET III

EXERCISE III.1 : NODES VOLTAGES

A. First method: superposition theorem.

a) **EFFECT OF THE 1^ST^ SOURCE**

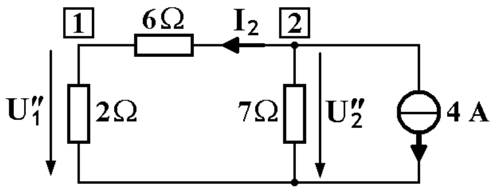


Voltage divider :

$$I_1 = \frac{2}{2+6+7}\cdot 1\,A = \frac{2}{15}\,A$$

$$\Rightarrow\ U_1' = (6+7)\cdot I_1 = \frac{26}{15}\,V \quad \text{et} \quad U_2' = 7\cdot I_1 = \frac{14}{15}\,V$$

b) **EFFECT OF THE 2^ND^ SOURCE**



Voltage divider :

$$I_2 = \frac{7}{15}\cdot(-4\,A) = -\frac{28}{15}\,A$$

$$\Rightarrow\ U_1'' = 2\cdot I_2 = -\frac{56}{15}\,V \quad \text{et} \quad U_2'' = (6+2)\cdot I_2 = -\frac{224}{15}\,V$$

c) **TOTAL VOLTAGES**

$$U_1 = U_1' + U_1'' = \frac{26}{15} - \frac{56}{15} = -2\,V \qquad \text{et} \qquad U_2 = U_2' + U_2'' = \frac{14}{15} - \frac{224}{15} = -14\,V$$

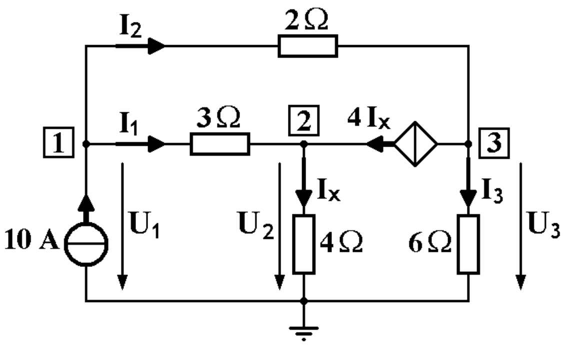
B. Second method: nodal analysis.

$$\begin{bmatrix} \frac{1}{2}+\frac{1}{6} & -\frac{1}{6} \\ -\frac{1}{6} & \frac{1}{6}+\frac{1}{7} \end{bmatrix}\cdot\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \end{bmatrix} \quad\Leftrightarrow\quad \begin{bmatrix} \frac{2}{3} & -\frac{1}{6} \\ -\frac{1}{6} & \frac{13}{42} \end{bmatrix}\cdot\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \end{bmatrix} \quad\Leftrightarrow\quad \begin{bmatrix} 28 & -7 \\ -7 & 13 \end{bmatrix}\cdot\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} 42 \\ -168 \end{bmatrix}$$

Cramer's method : $\Delta = 315$

$$U_1 = \frac{\begin{vmatrix} 42 & -7 \\ -168 & 13 \end{vmatrix}}{\Delta} = -2\,V \qquad U_2 = \frac{\begin{vmatrix} 28 & 42 \\ -7 & -168 \end{vmatrix}}{\Delta} = -14\,V$$

EXERCISE III.2 : NODES VOLTAGES



At node $\boxed{1}$: $10 = I_1 + I_2 = \dfrac{U_1 - U_2}{3} + \dfrac{U_1 - U_3}{2}$

$\Leftrightarrow \quad 60 = 2\cdot(U_1 - U_2) + 3\cdot(U_1 - U_3)$

$\Leftrightarrow \quad \boxed{5\,U_1 - 2\,U_2 - 3\,U_3 = 60} \qquad (1)$

At node $\boxed{2}$: $I_1 = -3\,I_x$

$\Leftrightarrow \quad \dfrac{U_1 - U_2}{3} = -3\cdot\dfrac{U_2}{4}$

$\Leftrightarrow \quad 4\,U_1 - 4\,U_2 = -9\,U_2$

$\Leftrightarrow \quad \boxed{4\,U_1 + 5\,U_2 = 0} \qquad (2)$

At node $\boxed{3}$: $I_2 = 4\,I_x + I_3$

$\Leftrightarrow \quad \dfrac{U_1 - U_3}{2} = 4\cdot\dfrac{U_2}{4} + \dfrac{U_3}{6}$

$\Leftrightarrow \quad 3\,U_1 - 3\,U_3 = 6\,U_2 + U_3$

$\Leftrightarrow \quad \boxed{3\,U_1 - 6\,U_2 - 4\,U_3 = 0} \qquad (3)$

Solving the system of equations (1), (2) et (3), we find :

$\boxed{U_1 = 80\text{ V}} \qquad \boxed{U_2 = -64\text{ V}} \qquad \boxed{U_3 = 156\text{ V}}$

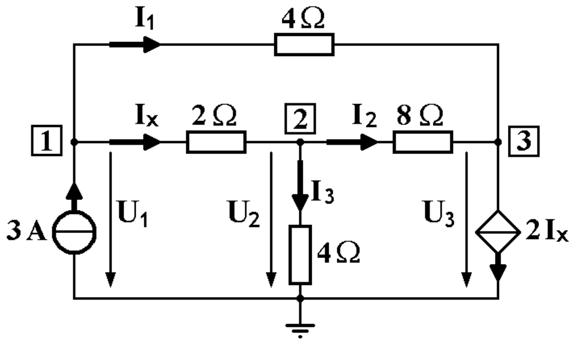
EXERCISE III.3 : MESH CURRENTS

$$\begin{bmatrix} 2+12+4 & -12 \\ -12 & 9+3+12 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix} \quad \Leftrightarrow \quad \begin{bmatrix} 18 & -12 \\ -12 & 24 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -8 \end{bmatrix}$$

$\Delta = 288$

$$I_1 = \frac{\begin{vmatrix} 12 & -12 \\ -8 & 24 \end{vmatrix}}{\Delta} = 0{,}67\text{ A} \qquad I_2 = \frac{\begin{vmatrix} 18 & 12 \\ -12 & 8 \end{vmatrix}}{\Delta} = 0\text{ A}$$

EXERCISE III.4 : NODES VOLTAGES



At node $\boxed{1}$:

$$3 = I_1 + I_X \quad \Rightarrow \quad 3 = \frac{U_1 - U_3}{4} + \frac{U_1 - U_2}{2}$$

$$\Rightarrow \quad 3\,U_1 - 2\,U_2 - U_3 = 12 \qquad (1)$$

At node $\boxed{2}$:

$$I_X = I_2 + I_3 \quad \Rightarrow \quad \frac{U_1 - U_2}{2} = \frac{U_2 - U_3}{8} + \frac{U_2}{4}$$

$$\Rightarrow \quad -4\,U_1 + 7\,U_2 - U_3 = 0 \qquad (2)$$

At node $\boxed{3}$:

$$I_1 + I_2 = 2\,I_X \quad \Rightarrow \quad \frac{U_1 - U_3}{4} + \frac{U_2 - U_3}{8} = \frac{2\,(U_1 - U_2)}{2}$$

$$\Rightarrow \quad 2\,U_1 - 3\,U_2 + U_3 = 0 \qquad (3)$$

The system of equations (1), (2) and (3) can be solved by two methods.

A. Algebraic method.

$$\begin{cases} 3\,U_1 - 2\,U_2 - U_3 = 12 \\ -4\,U_1 + 7\,U_2 - U_3 = 0 \\ 2\,U_1 - 3\,U_2 + U_3 = 0 \end{cases} \quad \Rightarrow \quad \boxed{U_1 = 4{,}8\ \text{V}} \quad \boxed{U_2 = 2{,}4\ \text{V}} \quad \boxed{U_3 = -2{,}4\ \text{V}}$$

B. Cramer's method.

$$\begin{bmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix} \qquad U_1 = \frac{\Delta_1}{\Delta} \qquad U_2 = \frac{\Delta_2}{\Delta} \qquad U_3 = \frac{\Delta_3}{\Delta}$$

$$\Delta = \begin{vmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{vmatrix} = 10$$

$$\Delta_1 = \begin{vmatrix} 12 & -2 & -1 \\ 0 & 7 & -1 \\ 0 & -3 & 1 \end{vmatrix} = 48 \qquad \Delta_2 = \begin{vmatrix} 3 & 12 & -1 \\ -4 & 0 & -1 \\ 2 & 0 & 1 \end{vmatrix} = 24 \qquad \Delta_3 = \begin{vmatrix} 3 & -2 & 12 \\ -4 & 7 & 0 \\ 2 & -3 & 0 \end{vmatrix} = -24$$

$$\Rightarrow \quad \boxed{U_1 = 4{,}8\ \text{V}} \quad \boxed{U_2 = 2{,}4\ \text{V}} \quad \boxed{U_3 = -2{,}4\ \text{V}}$$

EXERCISE III.5 : MESH ANALYSIS

Kirchhoff's law is applied to each mesh for the voltages.

In mesh I_1 :

$$-24+10\,(I_1-I_2)+12\,(I_1-I_3)=0 \quad \Rightarrow \quad 11\,I_1-5\,I_2-6\,I_3=12 \qquad (1)$$

In mesh I_2 :

$$-24\,I_2+4\,(I_2-I_3)+10\,(I_2-I_2)=0 \quad \Rightarrow \quad -5\,I_1+19\,I_2-2\,I_3=0 \qquad (2)$$

In mesh I_3 :

$$4\,I_0+12\,(I_3-I_1)+4\,(I_3-I_2)=0$$

But at node $\boxed{A}$, we have :

$$I_0=I_1-I_2$$

$$\Rightarrow \quad 4\,(I_1-I_2)+12\,(I_3-I_1)+4\,(I_3-I_2)=0 \quad \Rightarrow \quad -I_1-I_2+2\,I_3=0 \qquad (3)$$

The system of equations (1), (2) and (3) is written :

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix} \qquad I_1=\frac{\Delta_1}{\Delta} \qquad I_2=\frac{\Delta_2}{\Delta} \qquad I_3=\frac{\Delta_3}{\Delta}$$

$$\Delta=\begin{vmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{vmatrix}=192$$

$$\Delta_1=\begin{vmatrix} 12 & -5 & -6 \\ 0 & 19 & -2 \\ 0 & -1 & 2 \end{vmatrix}=432 \qquad \Delta_2=\begin{vmatrix} 11 & 12 & -6 \\ -5 & 0 & -2 \\ -1 & 0 & 2 \end{vmatrix}=144 \qquad \Delta_3=\begin{vmatrix} 11 & -5 & 12 \\ -5 & 19 & 0 \\ -1 & -1 & 0 \end{vmatrix}=288$$

$$\Rightarrow \quad I_1=2{,}25\text{ A} \qquad I_2=0{,}75\text{ A} \qquad I_3=1{,}5\text{ A} \quad \Rightarrow \quad \boxed{I_0=1{,}5\text{ A}}$$